Any open sentence that contains <, >, ≤, or ≥ is called an *inequality*.

→ has a variable

We often graph the solutions to inequalities on number lines. We put a small circle on the value, and the arrow points in the direction of the inequality, to the left for LESS THAN and to the right for GREATER THAN. Remember, for strictly less than (<) or strictly greater than (>), we use open circles to indicate that the value is NOT included in the solution. For less than or equal to (≤) or greater than or equal to (≥), we use closed circles to indicate that the value IS included in the solution.

○ = open circle

< or >

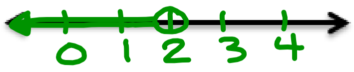
● = closed circle

≤ or ≥

EXAMPLES:

$x < 2$

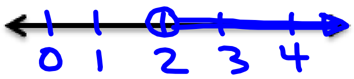
x is less than 2.



open circle because 2 is not less than 2.

$x > 2$

x is greater than 2.



open circle because 2 is not greater than 2.

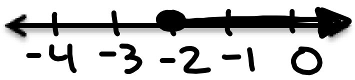
$x \le -2$

x is less than or equal to -2.

closed circle because -2 is less than or equal to -2.

$x \ge -2$

x is greater than or equal to -2.



closed circle because -2 is greater than or equal to -2.

KeyConcept Addition Property of Inequalities

Words If the same number is added to each side of a true inequality, the resulting inequality is also true.

Symbols For all numbers a, b, and c, the following are true.

1. If $a > b$, then $a + c > b + c$.
2. If $a < b$, then $a + c < b + c$.

$1407 > 1403$

$7 > 3 \rightarrow 7+1400 > 3+1400$

$3 < 7 \rightarrow 3+1400 < 7+1400$

$1403 < 1407$

* This property is also true for $\geq$ and $\leq$. *

KeyConcept Subtraction Property of Inequalities

Words If the same number is subtracted from each side of a true inequality, the resulting inequality is also true.

Symbols For all numbers a, b, and c, the following are true.

1. If $a > b$, then $a - c > b - c$.
2. If $a < b$, then $a - c < b - c$.

$-1595 > -1598$

$5 > 2 \rightarrow 5-1600 > 2-1600$

$2 < 5 \rightarrow 2-1600 < 5-1600$

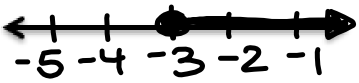
$-1598 < -1595$

* This property is also true for $\geq$ and $\leq$. *

✱ Solve the same way you would solve an equation.

Let's try solving some inequalities using the addition or subtraction properties. Use set-builder notation to show your solution set, and graph the solution set on a number line.

1. $8n \geq 7n - 3$



$$\begin{array}{rcl} 8n & \geq & \cancel{7n} - 3 \\ -7n & & -\cancel{7n} \\ \hline n & \geq & -3 \end{array}$$

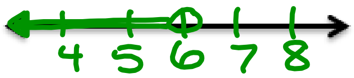
Set-builder notation:

$\{n \mid n \geq -3\}$

or

$\{n \geq -3\}$

2. $8r + 6 > 9r$



$$\begin{array}{rcl} \cancel{8r} + 6 & > & 9r \\ -\cancel{8r} & & -8r \\ \hline 6 & > & r \end{array}$$

$r < 6$

set-builder notation:

$\{r \mid r < 6\}$

or

$\{r < 6\}$

KeyConcept Multiplication Property of Inequalities

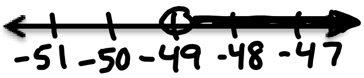
Words	Symbols	Examples
If both sides of an inequality that is true are multiplied by a positive number, the resulting inequality is also true. same process as with equations	For any real numbers a and b and any positive real number c, if $a > b$, then $ac > bc$. And, if $a < b$, then $ac < bc$.	$6 > 3.5$ $6(2) > 3.5(2)$ $12 > 7$ and $2.1 < 5$ $2.1(0.5) < 5(0.5)$ $1.05 < 2.5$
* * * If both sides of an inequality that is true are multiplied by a negative number, the direction of the inequality sign is reversed to make the resulting inequality also true.	For any real numbers a and b and any negative real number c, if $a > b$, then $ac < bc$. And, if $a < b$, then $ac > bc$.	$7 > 4.5$ $7(-3) < 4.5(-3)$ $-21 < -13.5$ and $3.1 < 5.2$ $3.1(-4) > 5.2(-4)$ $-12.4 > -20.8$

* This property also holds for inequalities involving $\leq$ and $\geq$. *

reciprocal: $\frac{3}{4} \rightarrow \frac{4}{3}$ flip the fraction.

Now let's try solving some inequalities using the multiplication property. Again, use set-builder notation to show your solution set, and graph the solution set on a number line.

3. $-\frac{3}{7}r < 21$



$$-\frac{3}{7}r < 21$$

$$-\frac{7}{3} \cdot -\frac{3}{7}r < -\frac{7}{3} \cdot \frac{21}{1}$$

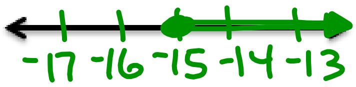
$$\frac{21}{21} = \quad 1r > -\frac{147}{3}$$

$$r > -49$$

Set-builder notation:

$\{r \mid r > -49\}$

4. $\frac{1}{5}m \geq -3$



$$\frac{1}{5}m \geq -3$$

$$\frac{5}{1} \cdot \frac{1}{5}m \geq 5 \cdot -3$$

$$1m \geq -15$$

$$m \geq -15$$

set-builder notation:

$\{m \mid m \geq -15\}$

KeyConcept Division Property of Inequalities

Words	Symbols	Examples
If both sides of a true inequality are divided by a positive number, the resulting inequality is also true.	For any real numbers a and b and any positive real number a, if $a > b$, then $\frac{a}{c} > \frac{b}{c}$. And, if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.	$4.5 > 2.1$, $\frac{4.5}{3} > \frac{2.1}{3}$, $1.5 > 0.7$ and $1.5 < 5$, $\frac{1.5}{0.5} < \frac{5}{0.5}$, $3 < 10$
* If both sides of a true inequality are divided by a negative number, the direction of the inequality sign is reversed to make the resulting inequality also true.	For any real numbers a and b, and any negative real number c, if $a > b$, then $\frac{a}{c} < \frac{b}{c}$. And, if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.	$6 > 2.4$, $\frac{6}{-6} < \frac{2.4}{-6}$, $-1 < -0.4$ and $-1.8 < 3.6$, $\frac{-1.8}{-9} < \frac{3.6}{-9}$, $0.2 > -0.4$

This property also holds true for inequalities involving ≤ and ≥.

Now let's try solving some inequalities using the division property. Again, use set-builder notation to show your solution set, and graph the solution set on a number line.

5. $-42 \geq 6r$

$$\frac{-42}{6} \geq \frac{\cancel{6}r}{\cancel{6}}$$

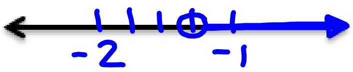
$$-7 \geq r$$

$$r \leq -7$$

set-builder notation:

$\{r \mid r \leq -7\}$

6. $-12h < 15$



$$\frac{\cancel{-12}h}{\cancel{-12}} < \frac{15}{-12}$$

$$h > -1.25$$

set-builder notation:

$\{h \mid h > -1.25\}$

word problems

Verbal problems containing phrases like *greater than* or *less than* can be solved by using inequalities. The chart shows some other phrases that indicate inequalities.

ConceptSummary Phrases for Inequalities

$<$	$>$	$\leq$	$\geq$
is less than fewer than	greater than is more than	at most, no more than, less than or equal to	at least, no less than, greater than or equal to

Define a variable, write an inequality to represent the problem and solve. Check your solution.

7. Felipe needs for the temperature of his leopard gecko's basking spot to be at least 82°F. Currently, the basking spot is 62.5°F. How much warmer does the basking spot need to be?

t = temperature

$$62.5 + t \geq 82$$
$$-62.5 \qquad -62.5$$
$$t \geq 19.5$$

$\boxed{\text{The temperature must be at least } 19.5^\circ \text{ warmer.}}$

8. Twice a number increased by 4 is at least 10 more than the number.

$+$

x= number

$$\begin{array}{r} 2x+4 \geq 1x+10 \\ -1x \downarrow \quad -1x \downarrow \\ \hline x+4 \geq 10 \\ -4 \quad -4 \\ \hline x \geq 6 \end{array}$$

The number is greater than or equal to 6.

9. Mario purchases a prepaid phone plan for \$50 at $\underbrace{\text{\$0.13 per minute}}$. How many minutes can Mario talk on this plan?

m = minutes

$$\frac{0.13m}{0.13} \leq \frac{50}{0.13}$$

$$m \leq 384.6153846$$

$$m \leq 384 \text{ minutes}$$

g = gallon

10. If gas costs $3.15 per gallon, how many gallons of gas, to the nearest tenth, can Jan buy for $24?

$$\frac{3.15g}{3.15} \leq \frac{24}{3.15}$$

$$g \leq 7.619047619$$

$$g \leq 7.6 \text{ gallons}$$

Jan can buy at most 7.6 gallons.